

Problems and Solutions Manual¹

to accompany

Derivatives: Principles & Practice

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Contents

Chapter 1. Futures & Options - Overview	3
Chapter 2. Futures Markets	7
Chapter 3. Pricing Forwards & Futures I	17
Chapter 4. Pricing Forwards & Futures II	32
Chapter 5. Hedging with Futures & Forwards	40
Chapter 6. Interest Rate Forwards & Futures	54
Chapter 7. Options Markets	69
Chapter 8. Option Payoffs & Trading Strategies	75
Chapter 9. No-Arbitrage Restrictions on Option Prices	85
Chapter 10. Early-Exercise & Put-Call Parity	97
Chapter 11. Option Pricing: An Introduction	107
Chapter 12. Binomial Option Pricing	123
Chapter 13. Implementing the Binomial Model	145
Chapter 14. Black & Scholes Model	155
Chapter 15. The Mathematics behind Black-Scholes	166
Chapter 16. Options Modeling: Beyond Black-Scholes	179
Chapter 17. Sensitivity Analysis: Greeks	193
Chapter 18. Exotic Options I: Path-Independent Options	207
Chapter 19. Exotic Options II: Path-Dependent Options	225
Chapter 20. Value at Risk	245
Chapter 21. Convertible Bonds	261
Chapter 22. Real Options	272
Chapter 23. Swaps and Floating Rate Products	279
Chapter 24. Equity Swaps	291
Chapter 25. Currency and Commodity Swaps	299

<i>Sundaram & Das: Derivatives - Problems and Solutions</i>	2
Chapter 26. Term Structure of Interest Rates: Concepts	309
Chapter 27. Estimating the Yield Curve	321
Chapter 28. Modeling Term Structure Movements	340
Chapter 29. Factor Models of the Term Structure	350
Chapter 30. The Heath-Jarrow-Morton and Libor Market Models	363
Chapter 31. Credit Derivative Products	378
Chapter 32. Structural Models of Default Risk	385
Chapter 33. Reduced-form Models of Default Risk	394
Chapter 34. Modeling Correlated Default	406
Chapter 35. Derivative Pricing with Finite-Differencing	417
Chapter 36. Derivative Pricing with Monte Carlo Simulation	421
appendix: Using Octave	431

Chapter 1. Futures & Options - Overview

1. **Question :** What is a derivative security?

Answer: A derivative security is a financial security whose value depends on (or derives from) other, more fundamental, underlying variables such as the price of a stock, a commodity price, an interest rate, an exchange rate, or an index level. The underlying may also be a derivative security itself – there are many derivatives that are written on other derivatives.

2. **Question :** Give an example of a security that is not a derivative.

Answer: An interest rate is not a derivative. It is a fundamental economic quantity reflecting the value of money.

A stock is also typically viewed as a “primitive” (rather than a derivative) security. However, a stock may also be viewed as a derivative: it represents a claim on the cash flows generated by the assets of the issuing firm. Indeed, viewing equity as a derivative written on the underlying firm’s asset value is fundamental to the “structural model” approach to studying corporate credit risk. We examine structural models in Chapter 32.

3. **Question :** Can a derivative security be the underlying for another derivative security? Give an example.

Answer: Yes, it can. The simplest example is an option (say, a call) that gives you the right to purchase another option (say, a put written on some underlying stock). In this case, the first option is called a “compound” option; it is an option on an option, in this case, a call on a put. Compound options are studied in Chapter 18. Compound options also play a role in the structural models studied in Chapter 32.

4. **Question :** Derivatives may be used for both hedging and insurance. What is the difference in these two motives?

Answer: The objective of hedging, whether with a derivative or otherwise, is to eliminate the risk associated with an existing market commitment and to create a net position that is “risk-free.” That is, the hedge nullifies existing risk; in so doing, it eliminates both upside and downside potential from market moves. Futures and forwards are instruments for hedging risk.

“Insurance” is one-sided protection. It guards against unfavorable events (“downside risk”) even while allowing full upside potential to be realized. Options are instruments that provide financial insurance.

5. **Question :** Define a forward contract. Explain at what time are cash flows generated for this contract. How is settlement determined?

Answer: A forward contract is an agreement to buy or sell an asset at a future date (denoted T), at a specified price called the delivery price (denoted F). Denote the initial date (the inception date or the date of the agreement) by $t = 0$. At inception there are no cash flows on a forward contract. At maturity, if the then-prevailing spot price S_T of the underlying asset is greater than F , then the buyer (the “long position”) has gained $S_T - F$ via the forward while the seller (the “short position”) has correspondingly lost $S_T - F$. Depending on contract specifications, the settlement may either be in cash (the seller pays the buyer $S_T - F$) or physical (the seller delivers the asset and receives F). If $S_T < F$, the buyer loses $F - S_T$ and the seller gains this quantity.

6. **Question :** Explain who bears default risk in a forward contract.

Answer: Default arises if, at maturity, one of the parties fails to fulfill their obligations under the contract. Default risk only matters for the party that is "in the money" at maturity, that is, that stands to profit at the locked-in price in the contract. (If the spot price at maturity is such that a party would lose from performing on the obligation in the contract, counterparty default is not a problem.) Prior to maturity, since either party may finish in-the-money, both parties are exposed to default risk.

7. **Question :** What risks are being managed by trading derivatives on exchanges?

Answer: An important one is counterparty default risk. In a typical futures exchange, the exchange interposes itself between buyer and seller and guarantees performance on the contract. This reduces significantly the default risk exposure of both parties. Further, daily settling of marked-to-market gains and losses ensures that the loss to the exchange from an investor's default is limited to at most one day's settlement amount (and because of maintenance margins is usually less than even this; see Chapter 2 for a description of the margining process).

8. **Question :** Explain the difference between a forward contract and an option.

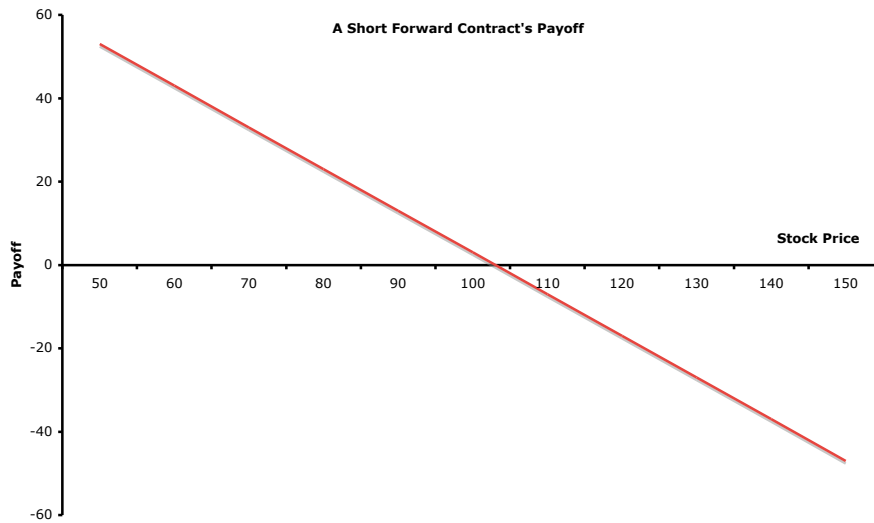
Answer: A forward contract is an agreement to buy or sell an asset at a future date (denoted T), at a specified delivery price (denoted F). The agreement is made at time $t = 0$ for settlement at maturity T . An option is the right but not the obligation to buy (a "call" option) or sell (a "put" option) an asset at a specified strike price on or before a specified maturity date T . In comparing a long forward contract to a call option, the main difference lies in the fact that the forward buyer has to buy the stock at the forward price at maturity, whereas in a call option, the buyer is not required to carry out the purchase if it is not in his interest to do so. The forward contract confers the obligation to buy, whereas the option contract provides this right with no attendant obligation.

9. **Question :** What is the difference between value and payoff in the context of derivative securities.

Answer: The *value* of a derivative is its current fair price or its worth. The *payoff* (or payoffs) refers to the *cash flows* generated by the derivative at various times during its life. For example, the value of a forward contract at inception is zero: neither party pays anything to enter into the contract. But the payoffs from the contract at maturity to either party could be positive, negative, or zero depending on where the spot price of the asset is at that point relative to the locked-in delivery price.

10. **Question :** What is a short position in a forward contract? Draw the payoff diagram for a short position at a forward price of \$103, if the possible range of the underlying stock price is \$50-150.

Answer: A short position in a forward is where you are the seller of the forward contract. In this case, you gain when the price of the underlying asset at maturity is below the locked-in delivery price. The payoff diagram for this contract is as shown in the following picture. When the price of the stock at maturity is the delivery price of \$103, there are neither gains nor losses.



11. **Question :** Forward prices may be derived using the notion of absence of arbitrage, and market efficiency is not necessary. What is the difference between these two concepts?

Answer: Absence of arbitrage means that a trading strategy cannot be found that creates cash inflows without any cash outflows, i.e., creates something out of nothing. Efficiency, as that term is used by financial economists, implies more: not only the absence of arbitrage but that asset prices reflect all relevant information.

12. **Question :** Suppose you are holding a stock position, and wish to hedge it. What forward contract would you use, a long or a short? What option contract might you use? Compare the forward versus the option on the following three criteria: (a) uncertainty of hedged position cash-flow, (b) Up-front cash-flow and (c) maturity-time regret.

Answer: If a forward contract is to be used, then a short forward is required. Alternatively, a put option may also be used. The following describes the three criteria for the choice of the forward versus the option.

- Cash-flow uncertainty is lower for the futures contract.
- The futures contract has no up-front cash-flow, whereas the option contract has an initial premium to be paid.
- There is no maturity-time regret with the option, because if the outcome is undesirable, the option need not be exercised. With the futures contract there is a possible downside.

13. **Question :** What derivatives strategy might you implement if you expected a bullish trend in stock prices? Would your strategy be different if you also forecast that the volatility of stock prices will drop?

Answer: If you expect prices to rise, there are several different strategies you could follow: you could go long a forward and lock in a price today for the future purchase; you could buy a call which gives you the right to buy the stock at a fixed strike price; or you could sell a put today, receive a premium, and keep the premium as your profit if prices trend upward as you expect.

The volatility issue is a bit trickier. As we explain in Chapter 7, both call and put options *increase* in value with volatility, so if you expect volatility to decrease, you do not want to *buy* a call: when volatility drops, what you have bought automatically becomes less valuable.

14. **Question :** What are the underlyings in the following derivative contracts?

- (a) A life insurance contract.
- (b) A home mortgage.
- (c) Employee stock options.
- (d) A rate lock in a home loan.

Answer: The underlyings are as follows:

- (a) A life insurance contract: the event of one's demise.
 - (b) A home mortgage: mortgage interest rate.
 - (c) Employee stock options: equity price of the firm.
 - (d) A rate lock in a home loan: mortgage interest rate.
-

15. **Question :** Assume you have a portfolio that contains stocks that track the market index. You now want to change this portfolio to be 20% in commodities and only 80% in the market index. How would you use derivatives to implement your strategy?

Answer: One would use futures to do so. We would short market index futures for 20% of the portfolio's value, and go long 20% in commodity futures. A collection of commodity futures adding up to the 20% would be required.

16. **Question :** In the previous question, how do you implement the same trading idea without using futures contracts?

Answer: Futures contracts are traded on exchanges and are known as "exchange-traded" securities. An alternative approach to achieving the goal would be to use an over-the-counter or OTC product, for example, an index swap that exchanges the return on the market index for the return on a broadly defined commodity index.

17. **Question :** You buy a futures contract on the S&P 500. Is the correlation with the S&P 500 index positive or negative? If the nominal value of the contract is \$100,000 and you are required to post \$10,000 as margin, how much leverage do you have?

Answer: The futures contract is positively correlated with the stock index. The leverage is 10 times. That is, for every \$1 invested in margin, you get access to \$10 in exposure.

Chapter 2. Futures Markets

1. **Question :** What are “delivery options” in a futures contract? Generally, why are delivery options provided to the short but not to the long position?

Answer: At settlement of a futures contract, the contract calls for the buyer to pay the seller the delivery price in exchange for the seller delivering to the buyer the specified grade of the underlying. Delivery options allow the short position to substitute an alternative grade or quality for the standard quality, at an adjustment in the delivery price. The futures contract specification lists the alternative deliverable grades and describes how the price will be adjusted for each grade.

Delivery options are provided because specifying the deliverable grade narrowly in a commodity futures contract may limit overall supply and facilitate market corners or squeezes. Corners and squeezes are market manipulation attempts in which the manipulator takes on more long positions in a given futures contract than the short position has ability to make delivery. This is achieved by the long either controlling all of the available spot supply (a “corner”) or at least a sufficient quantity so that the short position has difficulty finding adequate deliverable supply (a “squeeze”). In a successful attempt, the price of the commodity is driven up by the lack of supply. The short position must buy the required quantity for delivery at a high price and to sell it back to the long position at the fixed price agreed to in the contract (or equivalently must compensate the long position for the difference in prices). The provision of delivery options reduces the opportunity for such behavior by the long position. For exactly the same reason, delivery options are provided only to the short position and not the long.

2. **Question :** How do delivery options affect the relationship of futures prices to forward prices?

Answer: The delivery option is an option available only to the short position. The profit opportunity presented by delivery options to the short position comes at the expense of the long position. Other things being equal, the presence of delivery options means that the futures price will be lower than the forward price for a contract written on the standard grade. The presence of delivery options makes the futures contract more attractive to the short (who cannot lose from having this extra option), but less attractive to the long. With fewer buyers (long positions) and more sellers (short positions), the futures price will be lower than the forward price.

3. **Question :** To what do the following terms refer: *initial margin*, *maintenance margin*, and *variation margin*?

Answer: An investor opening a futures account is required to deposit a specified amount of cash into an account called the margin account. The amount deposited initially is called the *initial margin*.

At the end of each day, the balance in the margin account is adjusted to reflect the investor gains and losses from futures price movements over the day. This process is called marking-to-market. The changes to the margin account are called *variation margin*.

A critical minimum balance amount, called the *maintenance margin* is specified for the margin account. If the balance in the margin account falls below this level, then the investor receives a margin call requiring the account to be topped up back to the level of the initial margin; if the top-up does not occur, the account is closed out.

4. **Question :** What are price ticks?

Answer: Exchanges typically place restrictions on the minimum amount by which prices may change in either direction. These are known as price ticks and vary from contract to contract. In the US, the price tick is usually around \$10-\$25 per contract.

5. **Question :** Explain price limits and why they exist.

Answer: Exchanges often place restrictions on the maximum amount by which prices may fluctuate during a trading session. These are known as price limits. Price limits exhibit considerable variation across contracts ranging from around \$1,000 per contract in some cases to over \$10,000 per contract in others. In yet other cases, price limits may not exist at all, or may not be hard limits, operating, instead, in a flexible manner.

Price limits exist for the same reason as circuit breakers on the NYSE. They are aimed at preventing panics in the market, and are a function of the usual levels of volatility for the asset underlying the contract.

6. **Question :** What are position limits in futures markets? Why do we need these? Are they effective for the objective you state, or can you think of better ways to achieve the objective?

Answer: To reduce the possibility of market disruption and to prevent any single trader from exercising undue influence on prices, exchanges and regulators limit the maximum number of speculative positions an investor may hold. These are called position limits. Position limits vary from contract to contract. An important determinant of the limits is the likely physical supply of the underlying commodity. Most commodity futures contracts involve position limits, whereas in markets where supply is not a constraint (e.g., Treasury or currency futures), there are often no position limits. Also, investors who qualify as bonafide hedgers do not normally face position limits, though in practice this may mean that they are allowed much higher limits than speculators.

Position limits in single contracts may not be effective in controlling overall counterparty risk, since they do not account for the other aspects of the trader's portfolio which may contain risk-enhancing or mitigating positions, depending on the correlation of the various components of the portfolio. Other measures such as Value-at-Risk (to be studied in Chapter 20) may be more useful as they reflect the correct economic risk in contracts are usually better ways of managing total risk.

7. **Question :** What are the different ways in which futures contracts may be settled? Explain why these exist.

Answer: Futures contracts may be settled (a) by delivery, (b) in cash, and (c) by exchange of physicals.

The most common is physical delivery, where the short position actually delivers the asset to the long. For contracts settled by physical delivery, delivery may be effected on any day during the delivery month.

For some financial futures contracts, especially where physical delivery is a problem (stock index futures are an example), cash settlement is used. In cash settlement, the two sides simply settle the change in contract value in cash terms. For a numerical example of how cash settlement works, consider a COMEX gold futures contract. (At the time of writing, COMEX gold futures are physically settled; we assume cash settlement only to illustrate the computations.) Suppose you enter into a long gold futures contract at a futures price of \$930 per oz and the futures price at maturity is \$964 per oz. Then, by buying at \$930, you have gained an amount of $964 - 930 = 34$ per oz. If the contract is cash settled, you will receive \$34 per oz in cash from the short futures position. Cash settlement takes place through the margin account.

Finally, a third method called exchange of physicals is also possible. This is where long and short positions with equal position sizes negotiate price and delivery terms off-exchange, and communicate the details to the exchange. There are some restrictions on how EFPs may take place; in particular, they must involve physical delivery.

8. **Question :** What is meant by *open interest*?

Answer: The open interest measures the number of futures positions that have not yet been reversed.

9. **Question :** Discuss the liquidity and maturity of futures contracts.

Answer: Liquidity is, in general, the ease of getting in and out of a contract. In general, most futures contracts are highly liquid at short maturities but liquidity dries up as maturity increases. One measure of liquidity for futures contracts (but not an infallible one) is the size of open interest in that contract; a high open interest indicates a large number of participants and so a relatively liquid contract.

10. **Question :** Describe the standard bond in the Treasury Bond futures contract on the CBoT and the delivery option regarding coupons.

Answer: The standard bond in the Treasury bond futures contract is one with a face value of \$100,000, at least 15 years to maturity or first call, and a coupon of 6%.

Of the delivery options provided in the contract, the most important is the “quality option” that allows the short position to substitute any coupon for the standard 6%. The price that the long position has to pay is the quoted futures price times a conversion factor which depends on the bond that is actually delivered. The conversion factor is calculated by discounting the cash flows from the delivered bond at the standard 6% rate. If the delivered bond (a) has a coupon equal to the standard 6%, the conversion factor will be equal to one, since we are then discounting 6% cash flows at a 6% rate; (b) has a coupon higher than the standard 6%, the conversion factor will be greater than one; (c) has a coupon less than the standard 6%, the conversion factor will be less than one. See Section 2.4 and Chapter 6 for details on how the conversion factor is constructed.

11. **Question :** Suppose the delivered bond in the Treasury Bond futures contract has a remaining maturity of 20 years and a 7% coupon. Assume the last coupon was just paid. What is its conversion factor?

Answer: The conversion factor is

$$\frac{1}{100} \left[\frac{3.5}{1.03} + \dots + \frac{3.5}{1.03^{40}} + \frac{100}{1.03^{40}} \right] = 1.1156$$

12. **Question :** Suppose there are two deliverable bonds in the Treasury Bond futures contract, a 15-year 8% coupon bond and a 22-year 8% coupon bond. Assume the last coupon on both bonds was just paid. Which bond has the higher conversion factor? (Guess the answer first and then verify it by computation.)

Answer: Exercise for the reader. (Some exercise solutions are left to the reader intentionally. It fosters reading the text in some detail.)
